Answers to test yourself questions

Option D

D1 Stellar quantities

1 The distance is $\frac{1}{0.285} = 3.51$ pc.

2 The distance is $\frac{10.8}{3.26} = 3.31$ pc and so the parallax angle is $\frac{1}{3.31} = 0.302''$.

3 **a** The distance is $\frac{1}{0.0067} = 149$ pc.

b The diameter is

$$D = d\theta = 149 \times \frac{0.016}{3600} \times \frac{\pi}{180} \text{ pc} = 149 \times \frac{0.016}{3600} \times \frac{\pi}{180} \times 3.26 \times 9.46 \times 10^{15} = 3.56 \times 10^{11} \text{ m}.$$ The radius is

$\frac{3.56 \quad 10^{11}}{2} = 1.78 \quad 10^{11}$ m. This is about 256 times larger than the radius of the sun.

4 From Topic 12 we know that the nuclear radius for a nucleus of mass number A is given by $1.2 \times A^{1/3} \times 10^{-15}$ m. The nuclear volume is then

$$\begin{aligned} V &= \frac{4\pi}{3} R^3 \\ &= \frac{4\pi}{3}(1.2 \times A^{1/3} \times 10^{-15})^3 \\ &= 7.24 \times 10^{-45} \times A \ \text{ m}^3 \end{aligned}$$

The mass of the nucleus is A u, i.e. $A \times 1.66 \times 10^{-27}$ kg. The density is therefore (note how A cancels out)

$$\begin{aligned} \rho &= \frac{\cancel{A} \times 1.66 \times 10^{-27}}{7.24 \times 10^{-45} \times \cancel{A}} \\ &\approx 2.3 \times 10^{17} \text{ kg m}^{-3} \end{aligned}$$

So all nuclei have the same density and that density is comparable to a neutron star density.

5 The diameter d will be approximately equal to $d = D\theta$ where $D = 1.5 \quad 10^{11}$ m is the earth – sun distance and θ is the angle subtended by the sunspot in radians. Hence, $d = 1.5 \times 10^{11} \times \frac{4}{3600} \times \frac{\pi}{180} = 3 \times 10^6$ m.

6 The diameter d will be approximately equal to $d = D\theta$ where $D = 3.8 \quad 10^8$ m is the earth – moon distance and θ is the angle subtended in radians. Hence, $d = 3.8 \times 10^8 \times \frac{0.05}{3600} \times \frac{\pi}{180} = 92 \approx 10^2$ m.

7 The speed of the sun is $v = \frac{2\pi r}{T} = \frac{2\pi \times 2.8 \times 10^4 \times 9.46 \times 10^{15}}{211 \times 10^6 \times 365 \times 24 \times 3600} \approx 2.5 \times 10^5 \text{ m s}^{-1}$. Now using

$v^2 = \frac{GM}{r} \Rightarrow M = \frac{v^2 r}{G}$, we find $M = \frac{(2.5 \times 10^5)^2 \times 2.8 \times 10^4 \times 9.46 \times 10^{15}}{6.67 \times 10^{-11}} = 2.5 \times 10^{41}$ kg. This is the mass that is enclosed within a radius of 28000 ly. The mass in the galaxy outside this radius does not influence the motion of the sun.

8 **a** See discussion in textbook.

b The method fails for stars far away (more than about 300 pc or 1000 ly) because then the parallax angle is too small to be measured accurately.

9 We use $b = \dfrac{L}{4\pi d^2} \Rightarrow L = b \times 4\pi d^2$ so that $L = 3.0\times10^{-8} \times 4\pi \times (70\times9.46\times10^{15})^2$ i.e. $L = 1.7 \quad 10^{29}$ W.

10 We use $b = \dfrac{L}{4\pi d^2}$ so that $b = \dfrac{4.5\times10^{28}}{4\pi\times(88\times9.46\times10^{15})^2} = 5.2\times10^{-9}$ W m^{-2}.

11 From $b = \dfrac{L}{4\pi d^2}$ we get $d = \sqrt{\dfrac{L}{4\sigma b}}$, i.e. $d = \sqrt{\dfrac{6.2\times10^{32}}{4\pi\times8.4\times10^{-10}}} = 2.4\times10^{20}$ m. This corresponds to $\dfrac{2.4\times10^{20}}{9.46\times10^{15}} \approx 26$ kly.

12 **a** From $L = \sigma A T^4$, $\dfrac{L_H}{L_C} = \dfrac{\sigma A(4T)^4}{\sigma A T^4} = 4^4 = 256$.

b $$\frac{b_H}{b_C} = \frac{\left(\dfrac{L_H}{4\pi d_H^2}\right)}{\left(\dfrac{L_C}{4\pi d_C^2}\right)} \Rightarrow 1 = \frac{L_H}{L_C}\times\frac{d_C^2}{d_H^2} = 256\frac{d_C^2}{d_H^2} \text{ and so } \frac{d_C^2}{d_H^2} = \frac{1}{256} \Rightarrow \frac{d_C}{d_H} = \frac{1}{16}$$

13 $$\frac{b_A}{b_B} = \frac{\left(\dfrac{L_A}{4\pi d^2}\right)}{\left(\dfrac{L_B}{4\pi d^2}\right)} \Rightarrow \frac{9.0\times10^{-12}}{3.0\times10^{-13}} = \frac{L_A}{L_B}, \text{ i.e. } \frac{L_A}{L_B} = 30.$$

14 **a** Since $L = \sigma A T^4 = \sigma 4\pi R^2 T^4$: (a) $1 = \dfrac{R_A^2(5000)^4}{R_B^2(10000)^4} \Rightarrow \dfrac{R_A}{R_B} = \sqrt{\dfrac{(10000)^4}{(5000)^4}} = 4$.

b $$\frac{4.7\times10^{27}}{3.9\times10^{26}} = \frac{R_{star}^2(9250)^4}{R_{sun}^2(6000)^4} \Rightarrow \frac{R_{star}}{R_{sun}} = \sqrt{\frac{4.7\times10^{27}}{3.9\times10^{26}}\times\frac{(6000)^4}{(9250)^4}} \approx 1.5$$

15 **a** Since $L = \sigma A T^4 = \sigma 4\pi R^2 T^4$:

$$\frac{5.2\times10^{28}}{3.9\times10^{26}} = \frac{R_{star}^2(4000)^4}{R_{sun}^2(6000)^4} \Rightarrow \frac{R_{star}}{R_{sun}} = \sqrt{\frac{5.2\times10^{28}}{3.9\times10^{26}}\times\frac{(6000)^4}{(4000)^4}} \approx 26$$

b $$\frac{b_A}{b_B} = \frac{\left(\dfrac{L_A}{4\pi d_A^2}\right)}{\left(\dfrac{L_B}{4\pi d_B^2}\right)} \text{ and so } 2 = \frac{d_B^2}{d_A^2} \Rightarrow \frac{d_A}{d_B} = 0.71.$$

16 We have that $b = \dfrac{L}{4\pi d^2}$ and $L = \sigma A T^4$. Combining, $b = \dfrac{\sigma A T^4}{4\pi d^2}$.

Hence, $\dfrac{b_A}{b_B} = \dfrac{\left(\dfrac{\sigma A T_A^4}{4\pi d_A^2}\right)}{\left(\dfrac{\sigma A T_B^4}{4\pi d_B^2}\right)} = \dfrac{\left(\dfrac{T_A^4}{d_A^2}\right)}{\left(\dfrac{T_B^4}{d_B^2}\right)} = \dfrac{T_A^4 d_B^2}{T_B^4 d_A^2} \Rightarrow \dfrac{T_A}{T_B} = \sqrt[4]{\dfrac{b_A}{b_B}\dfrac{d_A^2}{d_B^2}}$. Since this is a ratio we do not have to change units (light years to meters.) Hence, $\dfrac{T_A}{T_B} = \sqrt[4]{\dfrac{8.0\times10^{-13}}{2.0\times10^{-15}}\dfrac{120^2}{150^2}} = 4$.

17 The distance to the star is $\dfrac{1}{0.034} = 29.4$ pc $= 29.4 \quad 3.09 \quad 10^{16} = 9.08 \quad 10^{17}$ m. The apparent brightness is then $b = \dfrac{L}{4\pi d^2} = \dfrac{2.45\times10^{28}}{4\pi\times(9.08\times10^{17})^2} = 2.4\times10^{-9}$ W m^{-2}.

D2 Stellar characteristics and stellar evolution

18 Light emitted from the star will have to pass through the outer layers of the star. Atoms in these layers may absorb light of certain wavelengths if these wavelengths correspond to energy differences in the atomic energy levels. The absorbed photons will therefore not make it through the outer layers of the star and will appear as dark lines in the spectrum of the star.

19 The dark lines in the absorption spectrum of a star indicate that photons of a wavelength corresponding to the dark lines have been absorbed by atoms in the outer layers of the star. Different atoms absorb different wavelength photons and so the dark lines are indicative of the type of elements present in the star.

20 The color of the star corresponds to a particular wavelength. This is the peak wavelength in the spectrum which in turn is related to surface temperature through Wien's law, $\lambda T = 2.9\times 10^{-3}$ K m.

21 We know that $L = \sigma A T^4$ and so $\dfrac{L_A}{L_B} = \dfrac{\sigma A_A T_A^4}{\sigma A_B T_B^4}$. Since the radius of A is double that of B, $\dfrac{L_A}{L_B} = 4\ \dfrac{T_A^4}{T_B^4}$. From Wien's law, $\lambda T = const$ and so $650\times T_A = 480\times T_B \Rightarrow \dfrac{T_A}{T_B} = \dfrac{480}{650}$. Hence, $\dfrac{L_A}{L_B} = 4\times\left(\dfrac{480}{650}\right)^4 = 1.2$.

22 **a** An HR diagram is a plot of the luminosity of star versus its surface temperature. Temperature is plotted increasing to the left on the horizontal axis.

b Such a plot reveals that stars are grouped into large classes: the main sequence which occupies a diagonal strip from top right to bottom left; the white dwarfs in the lower left corner and the red giants and supergiants in the top right corner.

c **i** The luminosity is $L = 10^4 L_\odot = 3.9\quad 10^{30}$ W and the radius is $R = 10R_\odot = 7.0\quad 10^9$ m. Therefore

$$3.9\times 10^{30} = 5.67\times 10^{-8}\times 4\pi(7.0\times 10^9)^2\times T^4$$

$$T = \left(\frac{3.9\times 10^{30}}{5.67\times 10^{-8}\times 4\pi(7.0\times 10^9)^2}\right)^{\frac{1}{4}}$$

$$T \approx 18000\text{ K}$$

ii From the mass luminosity relation $\dfrac{L}{L_\odot} = 10^4 = \left(\dfrac{M}{M_\odot}\right)^{3.5} \Rightarrow \dfrac{M}{M_\odot} = 10^{\frac{4}{3.5}} = 13.9$. Hence the density is

$$\rho = \frac{M}{V} = \frac{13.9M_\odot}{10^3 V_\odot} = 1.4\times 10^{-2}\rho_\odot$$

d Star B: the luminosity is the same as star A and the temperature is 3000 K. Hence

$$\frac{L_A}{L_B} = 1 = \frac{\sigma 4\pi R_A^2 T_A^4}{\sigma 4\pi R_B^2 T_B^4} = \frac{R_A^2 18000^4}{R_B^2 3000^4} \Rightarrow \frac{R_A}{R_B} \approx 2.8\times 10^{-2}.\ \text{So } R_B = \frac{R_A}{2.8\times 10^{-2}} = \frac{10R_\odot}{2.8\times 10^{-2}} = 360R_\odot.$$

Star C: the luminosity is $L = 10^{-3}L_\odot$ and the temperature is the same as that of star A. Hence

$$\frac{L_A}{L_C} = \frac{10^4 L_\odot}{10^{-3}L_\odot} = 10^7 = \frac{\sigma 4\pi R_A^2 T^4}{\sigma 4\pi R_C^2 T^4} = \frac{R_A^2}{R_C^2} \Rightarrow \frac{R_A}{R_C} = \sqrt{10^7} = 3.2\times 10^3$$

$$R_C = \frac{R_A}{3.2\times 10^3} = \frac{10R_\odot}{3.2\times 10^3} = 3.2\times 10^{-3}R_\odot$$

23 The temperature of the star is found from Wien's law to be $T = \dfrac{2.9\times 10^{-3}}{2.42\times 10^{-7}} = 1.20\times 10^4$ K. From the HR diagram this corresponds to a luminosity of about 20 solar luminosities. Therefore

$$d = \sqrt{\frac{20\times 3.9\times 10^{26}}{4\pi\times 8.56\times 10^{-12}}} = 8.5\times 10^{18}\text{ m.}$$

24 From the mass luminosity relation $L \propto M^{3.5}$ it follows that

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5} = 15^{3.5} = 1.3 \times 10^4.$$

25 **a** From the mass luminosity relation $L \propto M^{3.5}$ it follows that $\frac{L}{L_\odot} = 4500 = \left(\frac{M}{M_\odot}\right)^{3.5} \Rightarrow \frac{M}{M_\odot} = 4500^{\frac{1}{3.5}} \approx 11.$

b If it was its luminosity should have been $\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5} = 12^{3.5} \approx 6000.$ The actual luminosity is 3200 times that of the sun so this star cannot be a main sequence star.

27 The luminosity is about 4000 solar luminosities and so

$$d = \sqrt{\frac{4000 \times 3.9 \times 10^{26}}{4\pi \times 3.45 \times 10^{-14}}} = 1.9 \times 10^{21} \text{ m}$$

28 **a** The peak wavelength is about $\lambda = 0.40 \times 10^{-6}$ m and so the surface temperature (form Wien's law) is

$$T = \frac{2.9 \times 10^{-3}}{0.40 \times 10^{-6}} \approx 7.2 \times 10^3 \text{ K}.$$

b From the H-R diagram the luminosity is about 5-8 times that of the sun.

29 The temperature (form Wien's law) is $T = \frac{2.9 \times 10^{-3}}{7 \times 10^{-7}} \approx 4 \times 10^3$ K.

30 **a** The speed is given by $v = \frac{2\pi R}{T} = 2\pi R f = 2\pi \times 30 \times 10^3 \times 500 = 3.1 \times 10^7 \text{ m s}^{-1}$.

b $\frac{3.1 \times 10^7}{3.0 \times 10^8} \approx 10\%$

31 A red giant forms out of a main sequence star when a certain percentage of the hydrogen of the star is used up in nuclear fusion reactions. The core of the star collapses and this releases gravitational potential energy that warms the core to sufficiently high temperatures for fusion of helium in the core to begin. The suddenly released energy forces the outer layers of the star to expand rapidly and to cool down. The star thus becomes a bigger but cooler star – a red giant.

32 **a** A planetary nebula refers to the explosion of a red giant star that ejects most of the mass of the star into space.

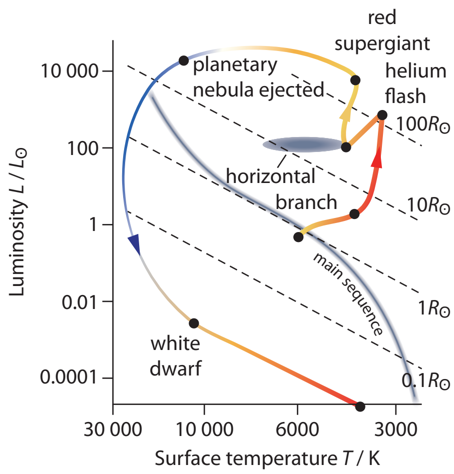
b Not all planetary nebulas (of the 3000 or so that are known to exist) appear as rings they way the famous helix and ring nebulas appear. The ring – like appearance is because the gas surrounding the center is very thin. A line of sight through the outer edges of the nebula goes through much more gas than a line of sight through the center. Hence the center looks transparent whole the edges do not.

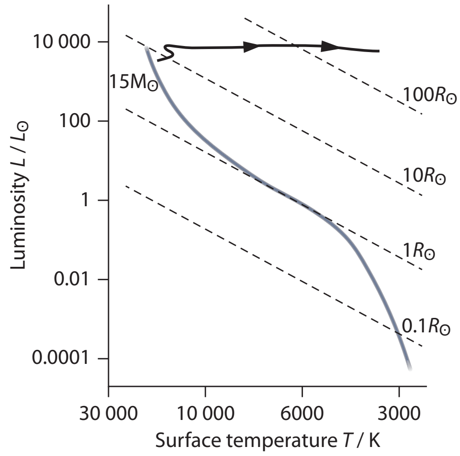
33 No, because all the elements that are necessary for life were made either in nuclear fusion processes in the cores of very heavy stars or during the supernova stage when nuclei were irradiated with neutrons (to make the elements heavier than iron).

34 **a** A 2 solar mass star would evolve to become a red giant. As the star expands in size into the red giant stage, nuclear reactions inside the core of the star are able to produce heavier elements than helium because the temperature of the core is sufficiently high. The red giant star will explode as a planetary nebula ejecting most of the mass of the star into space and leaving behind a dense core. The core is no longer capable of nuclear reactions and the star continues to cool down. The core is stable under further collapse because of electron degeneracy pressure. The core has a mass that is less than the Chandrasekhar limit and so ends up as a white dwarf.

b A 20 solar mass star would evolve to become a red supergiant. The red supergiant star will explode as a supernova ejecting most of the mass of the star into space and leaving behind a dense core. The core is no longer capable of nuclear reactions and the star continues to cool down. The core is stable under further collapse because of neutron degeneracy pressure. The core must have a mass that is less than Oppenheimer–Volkoff limit and so ends up as a neutron star.

c See graphs.





35 a A white dwarf forms as the core left behind after planetary nebula explosion of a red giant star.

b Whitedwarfs are very small (Earth size in radius) and very dense.

c A quantum mechanical principle known as the Pauli exclusion principle forbids neutrons to occupy the same quantum state. The enormous densities in neutrons stars try to force neutrons into the same state. A pressure develops among the neutrons to keep them apart and this balances the gravitational pressure.

36 Two from:

1. A main sequence star provides energy by nuclear fusion; no fusion takes place in a white dwarf.

2. With the exception of a few of the smallest main sequence stars (the red dwarfs) main sequence stars are larger in radius than a white dwarf.

3. The density of main sequence stars is much less than that of white dwarfs.

4. Main sequence stars are in equilibrium under the action of gravitational and radiation pressures whereas white dwarfs between gravitational and electron degeneracy pressures.

37 The density will be $\rho = \dfrac{1.0\times 10^{30}}{\dfrac{4\pi}{3}(6.4\times 10^{6})^{3}} = 9.1\times 10^{8}\ \text{kg m}^{-3}$.

38 Two from:

1. A main sequence star provides energy by nuclear fusion; no fusion takes place in a neutron star.

2. Even the smallest neutron star is larger in radius than a neutron star.

3. The density of main sequence stars is much less than that of neutron stars.

4. Main sequence stars are in equilibrium under the action of gravitational and radiation pressures whereas neutron stars between gravitational and neutron degeneracy pressures.

39 a A neutron star forms as the core left behind after a supernova in a red supergiant star.

b Neutron stars are very small (tens of km in radius) and very dense.

c A quantum mechanical principle known as the Pauli exclusion principle forbids neutrons to occupy the same quantum state. The enormous densities in neutrons stars try to force neutrons into the same state. A pressure develops among the neutrons to keep them apart and this balances the gravitational pressure.

40 This is mass of $1.4M_\odot$ and is the largest mass a white dwarf can have. A core with a mass larger than this limit cannot withstand the pressure of gravity by electron degeneracy pressure and will collapse further.

41 This is mass of $2-3M_\odot$ and is the largest mass a neutron star can have. A core with a mass larger than this limit cannot withstand the pressure of gravity by neutron degeneracy pressure and will collapse further, presumably without limit into a black hole.

(In 2011 the heaviest known neutron star was discovered using the National Radio Astronomy Observatory at Green Bank in Virginia in the US. The mass is $(1.97\pm 0.04)M_\odot$. The neutron star is a member of the binary pulsar J1614−2230. The significance of this discovery is that it rules out exotic forms of matter that have been proposed to exist inside neutron stars.)

42 The gas law states that $PV = nRT$. The number of moles is equal to $n = \dfrac{N}{N_A}$ where N_A is Avogadro's number. The Boltzmann constant k is defined by $k = \dfrac{R}{N_A}$ and so the gas law may be written as $PV = NkT$.

Since $V \propto R^3$ and $N \propto M$ we have that $PR^3 \propto MT$. From dimensional analysis, equilibrium demands that $PA \approx \dfrac{GM^2}{R^2}$ where A is the area on which the pressure P acts. Since $A \propto R^2$ it follows that $P \propto \dfrac{M^2}{R^4}$ and combining these two equations we get $T \propto \dfrac{M}{R}$. This shows that as the star shrinks (the radius gets smaller) the temperature goes up. Now, the luminosity is given by $L = \sigma AT^4 \propto R^2\left(\dfrac{M}{R}\right)^4 = \dfrac{M^4}{R^2}$. And since $\rho \propto \dfrac{M}{R^3}$ i.e. $R \propto \left(\dfrac{M}{\rho}\right)^{1/3}$ it follows that $L \propto \dfrac{M^4}{M^{2/3}} = M^{3.3}$, the mass-luminosity relation!

D3 Cosmology

43 a The distant galaxies move away from earth with a speed that is proportional to their distance from earth.

b This is evidence for the expanding universe because it implies that space in between galaxies is stretching meaning that the volume of the universe is increasing i.e. it expands.

44 No. These are nearby galaxies which show a blueshift because the mutual gravitational attraction between them and our Milky Way makes them move toward us.

45 Taking the Hubble constant to be $H = 68$ km s^{-1} Mpc^{-1} and Hubble's law, we find

$$v = Hd \Rightarrow d = \frac{v}{H} = \frac{500}{68} \approx 7 \text{ Mpc}.$$

46 There is no empty previously unoccupied space into which the galaxies are moving. Space is being created in between the galaxies.

47 The big bang signifies the beginning of time and space. At the big bang the universe was a point and so the big bang happened everywhere in the universe.

48 The question is meaningless *within the big bang model* since **by definition** time started with the big bang. It is as meaningless as to ask for a place 1 km north of the north pole. However, recent developments within string theory suggest that the question may not be as meaningless as it appears. See the very interesting article "The time before time", by Gabriele Veneziano (one of the true greats of theoretical physics) in the May 2004 Scientific American.

49 No, because these are nearby galaxies whose motion is much more affected by their mutual gravitational attraction rather than by the cosmic expansion.

50 a $$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0} \Rightarrow v = \frac{c\Delta\lambda}{\lambda_0} = \frac{3\times10^8 \times (658.9 - 656.3)}{656.3} \approx 1.2\times10^6 \text{ m s}^{-1}.$$

$$v = Hd \Rightarrow d = \frac{v}{H} = \frac{1.2\times10^6}{68\times10^3} \approx 18 \text{ Mpc}.$$ We have expressed the Hubble constant as $H = 72\times10^3$ m s^{-1} Mpc^{-1} so that the distance comes out in Mpc.

b $$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0} \Rightarrow v = \frac{c\Delta\lambda}{\lambda_0} = \frac{3\times10^8 \times (689.1 - 656.3)}{656.3} \approx 1.5\times10^7 \text{ m s}^{-1}.$$

$$v = Hd \Rightarrow d = \frac{v}{H} = \frac{1.5\times10^7}{68\times10^3} \approx 220 \text{ Mpc}\cdot$$

c $$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_0} \Rightarrow v = \frac{c\Delta\lambda}{\lambda_0} = \frac{3\times10^8 \times (704.9 - 656.3)}{656.3} \approx 2.2\times10^7 \text{ m s}^{-1}.$$

$$v = Hd \Rightarrow d = \frac{v}{H} = \frac{2.2\times10^7}{68\times10^3} \approx 320 \text{ Mpc}\cdot$$

d $\dfrac{v}{c} = \dfrac{\Delta\lambda}{\lambda_0} \Rightarrow v = \dfrac{c\Delta\lambda}{\lambda_0} = \dfrac{3\times10^8 \times (741.6 - 656.3)}{656.3} \approx 3.9\times10^7 \text{ m s}^{-1}$.

$v = Hd \Rightarrow d = \dfrac{v}{H} = \dfrac{3.9\times10^7}{68\times10^3} \approx 570 \text{ Mpc}$.

e $\dfrac{v}{c} = \dfrac{\Delta\lambda}{\lambda_0} \Rightarrow v = \dfrac{c\Delta\lambda}{\lambda_0} = \dfrac{3\times10^8 \times (789.7 - 656.3)}{656.3} \approx 6.1\times10^7 \text{ m s}^{-1}$.

$v = Hd \Rightarrow d = \dfrac{v}{H} = \dfrac{6.1\times10^7}{68\times10^3} \approx 900 \text{ Mpc}$.

51 **a** $v = Hd \Rightarrow d = \dfrac{v}{H} = \dfrac{3.0\times10^8}{68\times10^3} \approx 4.4\times10^3 \text{ Mpc}$.

b They are unobservable.

c No because the Hubble speed is not a real speed. It is the result of space in between galaxies stretching. It is not the speed of any material object.

52 The three standard pieces of evidence in favor of the big bang are (1) the cosmic background radiation, (2) the expansion of the universe and (3) the helium abundance in the universe.

53 **a** The redshift is $\dfrac{\Delta\lambda}{\lambda_0} = \dfrac{5.3\times10^{-7} - 4.5\times10^{-7}}{4.5\times10^{-7}} = 0.178 \approx 0.18$.

b $\dfrac{\Delta\lambda}{\lambda_0} = \dfrac{v}{c}$ so $v = 0.178c \approx 5.3\times10^4 \text{ km s}^{-1}$.

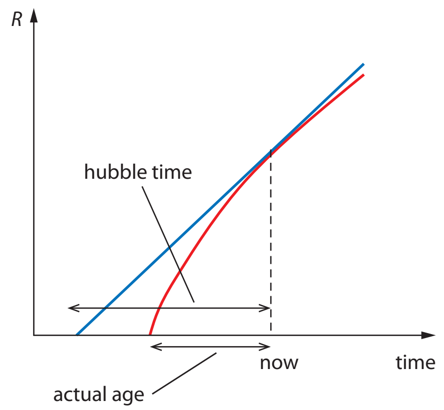
c From Hubble's law, $v = Hd$, we have that $d = \dfrac{v}{H} = \dfrac{5.3\times10^4 \text{ km s}^{-1}}{68 \text{ km s}^{-1}\text{ Mpc}^{-1}} \approx 780 \text{ Mpc}$.

54 At the time of the big bang the distance between any two objects was zero. A galaxy that today is at a distance d traveled this distance in time T, the age of the universe today and so $v = \dfrac{d}{T}$. The present speed of recession of the galaxy is $v = Hd$. Assuming that the galaxy had this speed throughout, then $\dfrac{d}{T} = Hd$, i.e. $T = \dfrac{1}{H}$.

With $H = 500 \text{ km s}^{-1}\text{ Mpc}^{-1}$, $T = \dfrac{1}{500 \text{ km s}^{-1}\text{ Mpc}^{-1}} = \dfrac{1}{500\times10^3} 10^6 \times 3.26 \times 9.46 \times 10^{15} \text{ s}$, i.e.

$T = 6.2 \quad 10^{19}$ s or 2 billion years. (The earth is older than this estimate.)

55 The estimate of the age of the universe as $T = \dfrac{1}{H}$ is based on the assumption that the galaxies have been moving away from earth at their present speed. The Hubble time is the age the Universe would have if it expanded at its present rate (blue line in the diagram below).

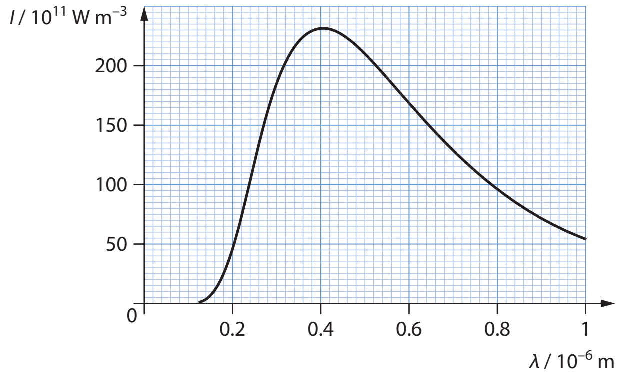


56 The fact that the speed of recession is proportional to the distance from earth implies that any other observer, anywhere else in the universe would reach the same conclusion, i.e. that he/she is at the center of the expansion. Thus, there is no center of expansion.

57 **a** The significance of the CMB is that it provides evidence for the Hot Big Bang theory. Radiation at a temperature of 2.7 k corresponds to a wavelength of about 1 mm. In a Hot Big Bang model radiation in the past would have been very high with a much smaller wavelength. As the Universe expands, space stretches and the wavelength of the radiation would increase as is observed.

b The same.

58 **a** See graph.



b $\lambda = \dfrac{2.9\times10^{-3}}{2.7} \approx 1.1\times10^{-3}$ m.

59 **a** It keeps decreasing approaching absolute zero.

b It would reach a minimum at the largest size of the universe and then would begin to increase as the universe begins to collapse.

60 **a** Redshift is the ratio of the difference between the observed and emitted wavelengths to the emitted wavelength.

b Space stretches in between galaxies and so does the wavelength,

c $z = \dfrac{v}{c} = \dfrac{H_0 d}{c} \Rightarrow d = \dfrac{cz}{H_0}$.

d $d = \dfrac{3.0\times10^{5}\times0.18}{68} \approx 790$ Mpc.

e $z = \dfrac{R}{R_0} - 1 \Rightarrow \dfrac{R}{R_0} = 1.18$.

61 **a** $d = \dfrac{cz}{H_0} = \dfrac{3.0\times10^{5}\times\dfrac{15}{486}}{68} \approx 140$ Mpc.

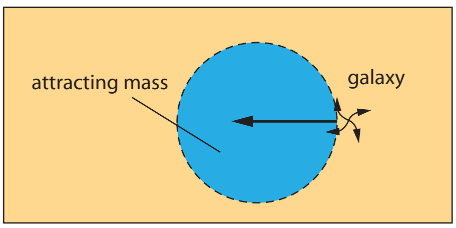
b $z = \dfrac{R}{R_0} - 1 = \dfrac{15}{486} \Rightarrow \dfrac{R}{R_0} = 1.03$

62 $z = \dfrac{R}{R_0} - 1 = \dfrac{1}{0.85} = 0.176$. Hence $d = \dfrac{cz}{H_0} = \dfrac{3.0\times10^{5}\times0.176}{68} \approx 780$ Mpc.

63 All type Ia supernovae have the same peak luminosity and so measuring its apparent brightness at the peak allows determining the distance.

64 **a** The speed of expansion of a distant galaxy is greater than what would have been assumed based on the simple Hubble law $v = Hd$.

b Gravity should have slowed them down.



c Distant Type Ia supernovae were used as standard candles i.e. objects of known luminosity. Measuring the apparent brightness at the peak allowed determination of distance. Measuring redshift allowed determination of the deceleration parameter of the universe because redshift and distance are related to each other though this parameter. High redshift values were needed and so very distant and very bright objects had to be used. Type Ia supernovae fitted these requirements.

d To determine the distance the apparent brightness had to be measured at a time when the luminosity was at the peak because the peak the luminosity was known.

65 In a decelerating universe the distance to a distant supernova would have been smaller and so we would expect to see a brighter supernova.

66 By systematically scanning large areas of the sky over and over again and obtaining very large numbers of digital images that could be analysed by a computer. In this way very large numbers of galaxies were examined.

D4 Stellar processes

67 A cloud of dust will collapse and form a star if the gravitational potential energy of the cloud molecules is greater than their random kinetic energy.

68 Once the gravitational potential energy of the clouds molecules is higher than the kinetic energy the cloud will begin to contract. This will release gravitational potential energy that will heat the cloud to temperatures large enough for nuclear fusion to take place.

69 Cold; so that the kinetic energy of the molecules will be smaller than their gravitational potential energy.

70 The mass luminosity relation states that $L \propto M^{3.5}$. This means that massive stars have a disproportionately high luminosity. In other words: a star will leave the main sequence when it exhausts a certain fraction k of its hydrogen mass by nuclear fusion. Then, roughly, $L = \frac{kM}{T}$ and so $M^{3.5} \propto \frac{kM}{T} \Rightarrow T \propto M^{-2.5}$ meaning that higher mass spend less time T on the main sequence.

71 From the previous problem, $T \propto M^{-2.5}$. Hence $\frac{T}{T_\odot} = \left(\frac{M}{M_\odot}\right)^{-2.5} = \frac{1}{2^{2.5}} = \frac{1}{5.7}$.

72 The energy produced in the sun's lifetime is $E = 3.9 \quad 10^{26} \quad 10^{10} \quad 365 \quad 24 \quad 3600 = 1.2 \quad 10^{44}$ J. This corresponds to a mass of $m = \frac{E}{c^2} = \frac{1.2 \quad 10^{44}}{9.0 \quad 10^{16}} = 1.3 \quad 10^{27}$ kg.

73 Because it marks the end of its life on the main sequence.

74 Once off the main sequence different sequences of nuclear fusion reactions take place depending on the mass of the star. The more massive the star the heavier the elements produced. The elements arrange themselves according to mass the heaviest being at the core and the lightest in the outer layers creating the onion like structure.

75 **a** On the main sequence the main nuclear reaction for low mass stars is the proton-proton cycle that produces helium by fusing hydrogen.

b A low mass star that leaves the main sequence will produce carbon in the triple alpha process in which three helium-4 nuclei produce carbon-12 (with the intermediate and subsequent fusing of beryllium).

76 **a** Such a massive star will produce helium both with the proton-proton cycle and the CNO cycle.

b After leaving the main sequence much heavier elements will be produced beginning with carbon in the triple alpha process and then neon, sodium, magnesium and silicon.

77 To produce elements by nuclear fusion requires the binding energy of the product nuclei to be higher than that of the reactants. Since iron is near the peak of the binding energy curve nothing heavier than iron can be produced by fusion.

78 **a** helium

b helium

c carbon

79 The s and r processes refer to neutron absorption by nuclei. Neutron absorption increases the mass number of a nucleus and the resulting isotope will in general decay by beta decay increasing the atomic number by 1 and thus producing a heavier element. In the slow s process, the isotope decays before the nucleus has time to absorb another neutron. In the fast r process the nucleus absorbs more than one neutron before it decays. The subsequent beta decay of the heavy isotope again produces heavier elements. The r process happens during a supernova explosion.

80 Heavier elements have higher atomic numbers and therefore higher positive electric charges. To fuse means that the nuclei have to come very close to each other and this requires higher kinetic energies. This in turn implies higher temperatures.

81 A Type Ia supernova is formed when a white dwarf that is in binary star system accretes mass from the companion star. The additional mass forces the white dwarf to exceed the Chandrasekhar limit and the star can no longer maintain equilibrium. The white dwarf contains mainly carbon and oxygen. The additional mass causes temperatures in the core to increase sufficiently for nuclear reactions to take place. The result is that the star blows up with enormous energy release.

82 A type II supernova is formed when a massive main sequence star that has evolved away from the main sequence and entered the red supergiant region explodes as supernova.

83 The main difference is their mechanism of production as described in the previous two problems. Additional differences include the shape of the light curve (i.e. the graph of luminosity versus time): the light curve for Type Ia falls off more sharply than that for Type II. Type II have hydrogen absorption lines whereas Type Ia do not.

84 The nuclear reactions inside massive stars produce an onion like structure with the heaviest elements at the core and the lightest, i.e. hydrogen, in the outer layers. Hence when the star goes supernova there is enough hydrogen present to produce hydrogen absorption lines.

85 Both have the net effect of turning 4 hydrogen nuclei into one nucleus of helium and both take place in main sequence stars. The CNO cycle requires higher temperatures and therefore takes place in more massive stars.

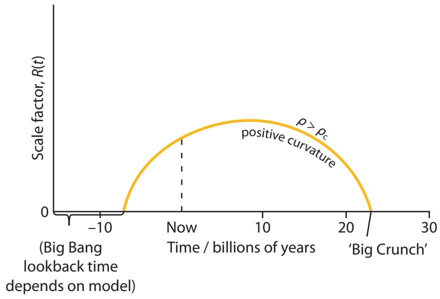
D5 Further cosmology

86 The cosmological principle states that the universe, on a large scale, is homogeneous(no one position is special) and isotropic (no one direction is special). This means that when viewed from different positions and different directions the observer sees the same distribution of matter and energy. **a** It can be used to deduce that the Universe has no centre for if it did observing from the centre would reveal a different picture than from any other point. **b** It can also be used to deduce that the Universe has no edge because, again, if it did viewing things from the edge would give a different picture than from appoint far from the edge.

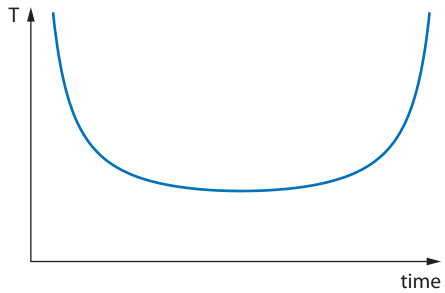
87 The main evidence for the cosmological principle is the isotropy of the CMB and the same distribution of galaxies in any direction from Earth. The fluctuations observed in the CMB are very small and so provide strong evidence for isotropy. Evidence for isotropy is also provided by the identical distribution of radio galaxies in different directions. Homogeneity is more difficult to test because we have not been able to move far from Earth so we have evidence only based in the small regions of the Universe that is our immediate neighbourhood.

88 **a** A factor that relates the physical distance between two points in space in terms and the coordinates of the points.

b See graph below.



c Since the temperature depends on the scale factor according to $T \propto \dfrac{1}{R}$ we have a graph like this.



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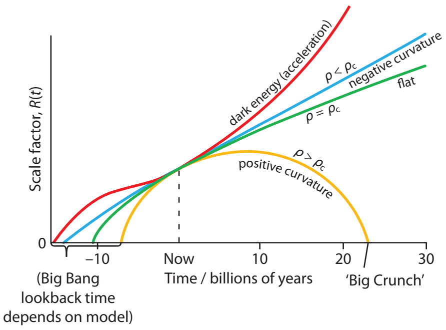
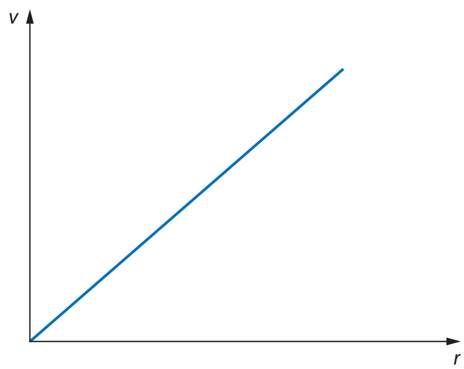


The graphs are arranged so that at the present time all models agree on the value of the Hubble constant because they have the same tangent line. The lines start at different times implying a different age of the universe depending on the model chosen.

90 **a** Consider a cloud of uniform density ρ and a point particle moving in a circular orbit of radius r about the centre of the cloud. The speed of the particle is given by $v = \sqrt{\dfrac{GM}{r}}$ where M is the mass of the cloud within a sphere of radius r. We have that $M = \rho V = \rho\dfrac{4\pi r^3}{3}$ and so $v = \sqrt{\dfrac{G\rho\dfrac{4\pi r^3}{3}}{r}} = \sqrt{G\rho\dfrac{4\pi r^2}{3}}$ i.e. $v \propto r$.

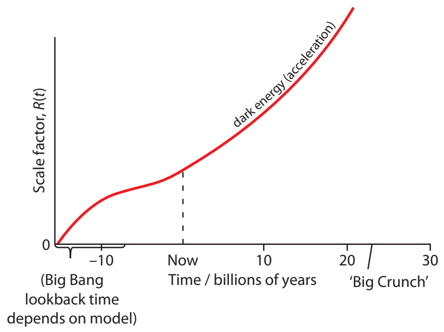
b See graph here.



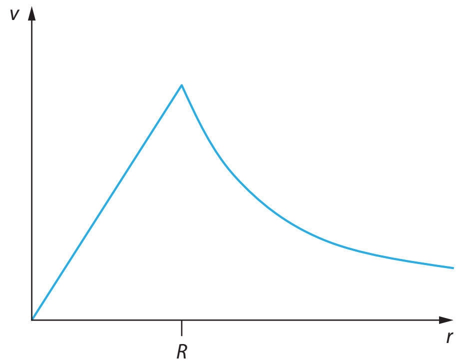
91 **a** The speed of the particle is given by $v = \sqrt{\dfrac{GM}{r}}$ and so $v = \sqrt{\dfrac{Gkr}{r}} = \sqrt{Gk}$ i.e. v = constant.

b Far from the center the rotation speed approaches a constant consistent with a distribution $M(r) = kr$. This implies that there is substantial mass at large values of r i.e. in the outer edges of the galaxy.

92 See graph here.



93 **a**



b In the graph above the curve approaches zero while in our galaxy it approaches a non zero constant.

94 **a** Dark matter is matter that is known to exist because of its gravitational effects on the motion of other nearby matter but is too cold to radiate and so cannot be seen.

b It could be baryonic matter i.e. matter consisting of ordinary protons and neutrons such as white, brown and black dwarfs as well as very small planets. However, baryonic matter by itself cannot account for all the dark matter that is known to exist. Other candidates include neutrinos and exotic particles predicted by supersymmetry.

95 Dark matter is mater that does not radiate and so cannot be seen. Dark energy is a vacuum energy that fills all space and is supposed to be responsible for a repulsive force that is accelerating the rate of expansion of the universe.

96 In an accelerating universe distant supernovae are further out than imagined and hence dimmer than what would be expected based on models in which the expansion slows down.

97 **a** The CMB radiation is isotropic to a very high degree which means that no matter in which direction one looks the spectrum of the CMB radiation is the same. However there are very small deviations from this perfect isotropy of the order of millionths of a kelvin: the temperature of the radiation is not constant but varies by these tiny amounts.

b The anisotropies in the CMB are important because they are needed in order to explain the formation of structures in the universe such as stars and galaxies. In addition studies of the anisotropy also give information about various cosmological parameters most importantly about the curvature of the universe.

98 The magnitude of the fluctuations in temperature fluctuations is a direct function of the geometry of the universe. The measured values are consistent with a flat universe.

99 According to D44 every model with a negative cosmological constant (i.e. $\Omega_{\Lambda} < 0$) involve a collapsing universe which is not the case.

100 By the Wien displacement law $\lambda T = \text{constant}$. The cosmological origin of redshift implies that $\lambda \propto R$ and so $T \propto \frac{1}{R}$.

101 **a** In a model with zero dark energy (i.e. cosmological constant Λ) this is the density that separates a universe that expands forever to a universe that will eventually collapse. A universe with $\Lambda = 0$ and a density equal to the critical density would expand forever but with a rate that approaches zero at infinity.

b The total energy of a mass m a distance r from the centre of the cloud is $E = \frac{1}{2}mv^2 - \frac{GMm}{r}$ where M is the mass of the cloud. If we call the density of this cloud ρ, then $M = \rho\frac{4}{3}\pi r^3$ and using this together with $v = Hr$ we find $E = \frac{1}{2}mr^2\left(H_0^2 - \frac{8\pi\rho G}{3}\right)$. The particle will move to infinity and just about stop if $E = 0$ i.e. at the critical density $\rho_c = \frac{3H^2}{8\pi G}$.

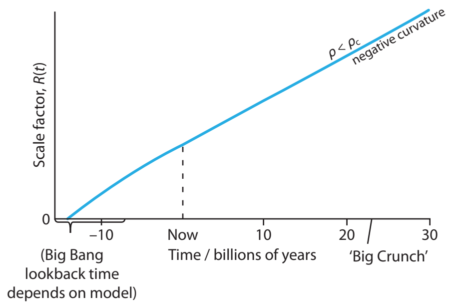
c The critical density is $\rho_c = \frac{3H^2}{8\pi G} = \frac{3\times\left(\frac{68\times10^3}{10^6\times3.09\times10^{16}}\right)^2}{8\pi\times6.67\times10^{-11}} = 8.7\times10^{-27} \approx 10^{-26}\ \text{kg m}^{-3}$. Hence the matter density of the universe is $\Omega_m = \frac{\rho}{\rho_c} \Rightarrow \rho = 0.32\times8.7\times10^{-27} = 3\times10^{-27}\ \text{kg m}^{-3}$.

102 **a** It is difficult because it involves estimating the mass in large volumes of space far from the Earth and this implies large uncertainties. In addition there is a lot of matter in the universe that does not radiate and so cannot be seen.

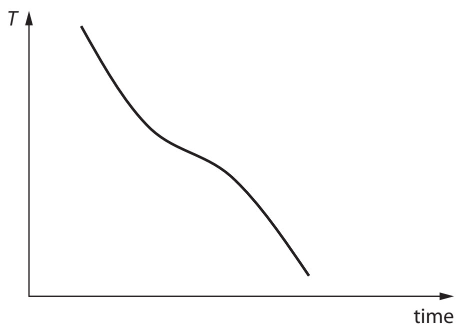
b In $1\ \text{m}^3$ we have a mass of 10^{-26} kg and so $\frac{10^{-26}}{1.67\times10^{-27}} \approx 6$ atoms of hydrogen.

103 **a** A model in which the presence of dark energy overcomes the retarding gravitational force making *the rate of change* of the scale factor to change a positive rate. The term refers specifically to a model with a positive cosmological constant or dark energy and flat curvature.

b See graph.



c See graph.



104 The surface of a flat sheet of paper extending forever is an example of what is called an open universe, whereas the surface of a sphere (just the surface not the interior) is an example of a closed universe. The surface of a sphere is finite (it has finite area) but it has no boundary – you cannot walk on a sphere and come to a point where you see an edge. On the other hand, a finite flat piece of paper is an example of a finite space (finite area) that does have an edge, a boundary.

105 Use $\rho_\Lambda = \dfrac{\Lambda c^2}{8\pi G} = \Omega_\Lambda \rho_c = 0.68 \times 10^{-26}\ \mathrm{kg\,m^{-3}}$. Hence

$$\Lambda = \frac{0.68 \times 10^{-26} \times 8\pi G}{c^2}$$

$$\Lambda = \frac{0.68 \times 10^{-26} \times 8\pi \times 6.67 \times 10^{-11}}{9.0 \times 10^{16}} \frac{(\mathrm{N\,kg^{-2}\,m^2})(\mathrm{kg\,m^{-3}})}{\mathrm{m^2\,s^{-2}}}$$

$$\Lambda = 1.3 \times 10^{-52} \approx 10^{-52}\ \mathrm{m^{-2}}$$

106 **a** The universe is known to expand so R varies. The redhift of light from distant galaxies implies an increasing R. The existence of the CMB implies an increasing R.

b It could be anywhere on the blue line separating the expanding and collapsing phases in the D44 depending on the values chosen for the matter density and the cosmological constant.

107 We know that $\rho_c = \dfrac{3H^2}{8\pi G}$, $\Omega_\mathrm{m} = \dfrac{\rho}{\rho_c}$ and $\Omega_\Lambda = \dfrac{\rho_\Lambda}{\rho_c}$ with $\rho_\Lambda = \dfrac{\Lambda c^2}{8\pi G}$. Substituting in the Friedmann equation $H^2 = \dfrac{8\pi G}{3}\left(\rho + \dfrac{\Lambda c^2}{8\pi G}\right) - \dfrac{kc^2}{R^2}$ gives

$$H^2 = \frac{8\pi G}{3}(\Omega_\mathrm{m} + \Omega_\Lambda)\rho_c - \frac{kc^2}{R^2}$$

$$H^2 = \frac{H^2}{\rho_c}(\Omega_\mathrm{m} + \Omega_\Lambda)\rho_c - \frac{kc^2}{R^2}$$

$$1 = (\Omega_\mathrm{m} + \Omega_\Lambda) - \frac{kc^2}{H^2R^2}$$

So if $\Omega_\mathrm{m} + \Omega_\Lambda = 1$ it follows that $k = 0$, a flat universe.